PHYSICS OF MATERIALS

Physics School Autumn 2024

Series 7 Solution **15 November 2024**

Exercise 1 Interaction force between edge dislocations

Two parallel edge dislocations are shown in the drawing of Fig. 7.1

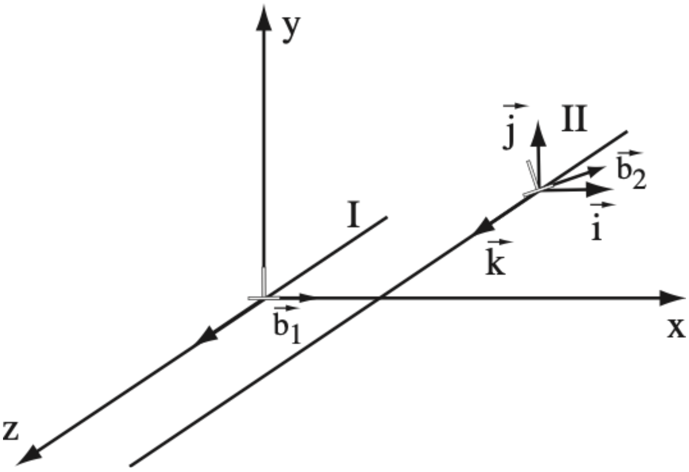

Fig. 7.1 Dislocation diagram

We chose the axis so that the two dislocations are parallel to Oz. Therefore:

$$\vec{b} \text{ of } I = b\vec{i}$$
$$\vec{b} \text{ of } II = b_1\vec{i} + b_2\vec{j}$$

The stress created by the dislocation *I* is given by the expressions (7.13). We thus have the force per unit length exerted by dislocation *I* on dislocation *II*:

$$\vec{f} = \begin{pmatrix} b_1 & b_2 & 0 \end{pmatrix} \cdot \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \sigma_{xx}b_1 + \sigma_{yx}b_2 & \sigma_{xy}b_1 + \sigma_{yy}b_2 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The force $\vec{f}$ thus has a component following $\vec{i}$: (f_x) and a component following $\vec{j}$: (f_y).

Using the formula (7.13) of the course:

$$\sigma_{xx} = -D\frac{\sin\theta(2+2\cos 2\theta)}{r} \qquad \sigma_{xy} = D\frac{\cos\theta\cos 2\theta}{r} \qquad \sigma_{yy} = D\frac{\sin\theta\cos 2\theta}{r}$$

with $D = \dfrac{\mu b}{2\pi(1-\nu)}$

We get the following:

$$f_x = \frac{D}{r}\left(b_1 \cos\theta \cos 2\theta + b_2 \sin\theta \cos 2\theta\right)$$
$$f_y = \frac{D}{r}\left(b_1 \sin\theta(2+\cos 2\theta) - b_2 \cos\theta \cos 2\theta\right)$$

If $b_2 = 0$, then:

$$f_x = \frac{\mu b_1 b}{2\pi(1-\nu)}\cdot\frac{\cos\theta \cos 2\theta}{r} \quad \text{and} \quad f_y = \frac{\mu b_1 b}{2\pi(1-\nu)}\cdot\frac{\sin\theta\ (2+\cos 2\theta)}{r}$$

The direction of these forces is indicated in Fig. 7.2:

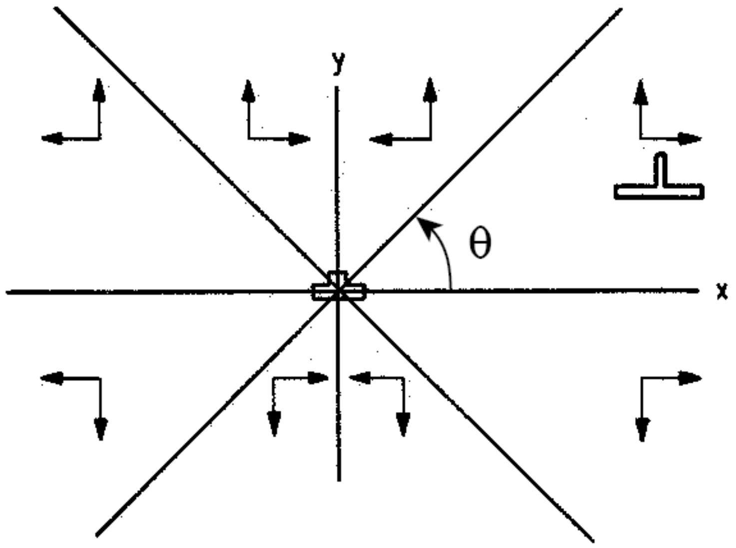

Fig. 7.2

This direction is reversed if the two dislocations have opposite signs. Please note that at room temperature, edge dislocations easily move only in their slip plane, and thus, the critical term is f_x. Let's see how f_x varies vs. x, which can be written in cartesian coordinates as:

$$f_x = \frac{\mu b_1 b}{2\pi(1-\nu)}\ \frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

We can easily see in this expression that if x is positive, f_x is negative if $x < y$ and vice versa if $x > y$. **The shape of this function (Fig. 7.3) allows a better understanding of how edge**

dislocations space and align themselves periodically to form misorientation-type grain boundaries (tilt boundary).

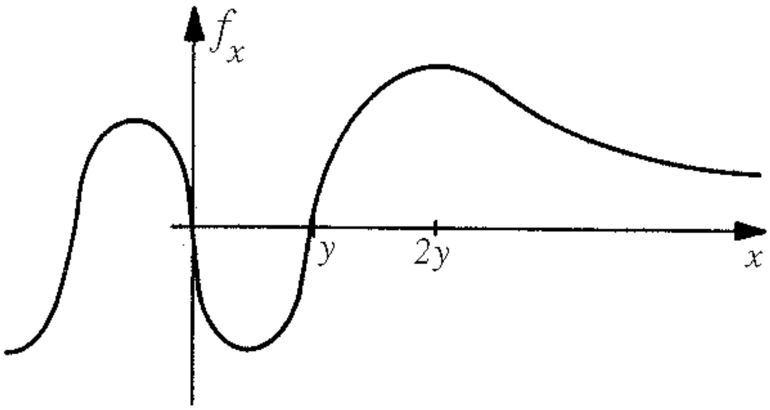

Fig. 7.3

Exercise 2 Interaction force between an edge and a screw dislocation

Figure 7.4 shows how to set the dislocations in the coordinate system. In the general case, the edge dislocation has a Burgers vector in the plane Ozy.

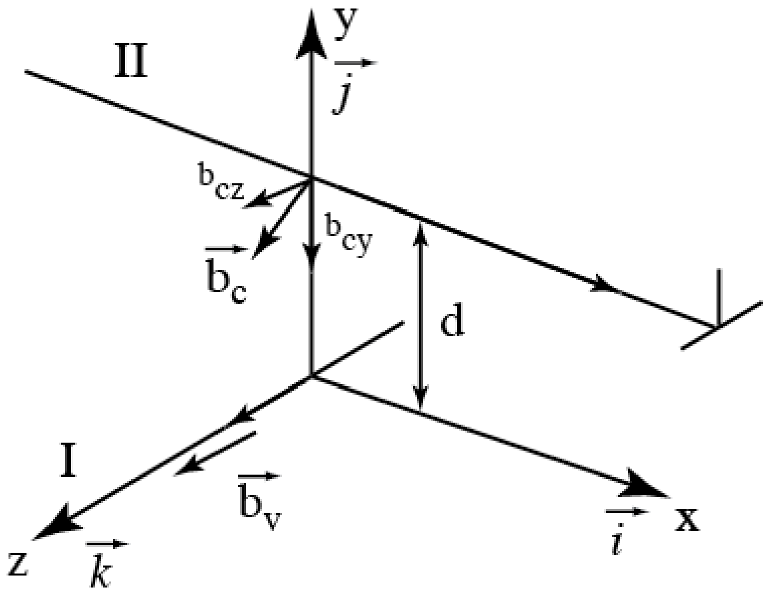

Fig. 7.4 Coordinate scheme

$\vec{b}$ of I (screw dislocation) $= b_v\vec{k}$

$\vec{b}$ of II (edge dislocation) $= b_{cy}\vec{j} + b_{cz}\vec{k}$

We use the expressions in Cartesian coordinates of the stress produced by a screw dislocation with infinite length (equation 7.5 in Chapter 7 text):

$$\sigma_v = \begin{pmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & 0 \end{pmatrix}$$ and thus $$\vec{b}_c \cdot \sigma_v = \begin{pmatrix} b_{cz}\sigma_{xz} \\ b_{cz}\sigma_{yz} \\ b_{cy}\sigma_{yz} \end{pmatrix}$$

We get the force caused by the screw dislocation on the edge dislocation:

$$\vec{f} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \sigma_{xz}b_{cz} & \sigma_{yz}b_{cz} & \sigma_{yz}b_{cy} \\ 1 & 0 & 0 \end{vmatrix} = \sigma_{yz}\, b_{cy}\, \vec{j} - \sigma_{yz}\, b_{cz}\, \vec{k}$$

$$\vec{f} = \frac{\mu b_v b_{cy}}{2\pi} \frac{x}{x^2+y^2}\ \vec{j} - \frac{\mu b_v b_{cz}}{2\pi} \frac{x}{x^2+y^2}\ \vec{k}$$

The edge dislocation undergoes a climb force along ***y*** and a slip force along **z.** To simplify the analysis, let's suppose that the dislocation stays in its slip plane at a distance d from the screw dislocation. Let's analyze the slip force supposing $b_{cy} = 0$. We note that the component along ***z*** changes sign depending on ***x***. Therefore, there is no net average force, but there is still a change in the edge dislocation line that follows the spiral distortion of atomic planes caused by the screw dislocation (Fig. 7.5).

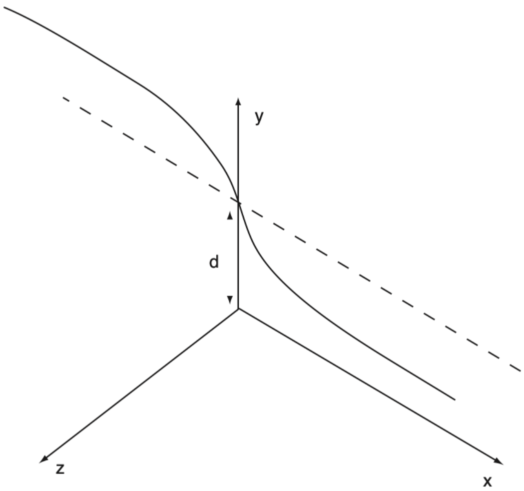

Fig. 7.5 Distortion of the edge dislocation

We can also calculate the force caused by the edge dislocation on the screw dislocation. However, to simplify the numeric result, we only consider the case where the Burgers vector of the edge dislocation is parallel to ***z***.

$\vec{b}$ of I (edge dislocation) $= b_c\vec{k}$
$\vec{b}$ of II (screw dislocation) $= b_v\vec{k}$

$$\sigma_c = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & \sigma_{yz} \\ 0 & \sigma_{yz} & \sigma_{zz} \end{pmatrix}$$ therefore $$\vec{b}_v \cdot \sigma_c = \begin{pmatrix} 0 \\ b_v\sigma_{yz} \\ b_v\sigma_{zz} \end{pmatrix}$$

$$\vec{f} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & b_v\sigma_{yz} & b_v\sigma_{zz} \\ 0 & 0 & 1 \end{vmatrix} = \sigma_{yz}\, b_v\, \vec{i}$$

$$\vec{f} = \frac{\mu b_v b_c}{2\pi(1-\nu)} \frac{\cos(\theta)\cos(2\theta)}{r} \cdot \vec{i} = -\frac{\mu b_v b_c}{2\pi(1-\nu)} \frac{\frac{y}{r}(\cos^2(\theta)-\sin^2(\theta))}{\sqrt{y^2+z^2}} \cdot \vec{i} =$$

$$= \frac{\mu b_v b_c}{2\pi(1-\nu)} \frac{\frac{y}{\sqrt{y^2+z^2}}\left(\frac{y^2}{y^2+z^2} - \frac{z^2}{y^2+z^2}\right)}{\sqrt{y^2+z^2}} = \frac{\mu b_v b_c}{2\pi(1-\nu)} \frac{y(y^2-z^2)}{(y^2+z^2)^2} \cdot \vec{i} \quad = -\frac{\mu b_v b_c}{2\pi(1-\nu)} \frac{d(z^2-d^2)}{(z^2+d^2)^2} \cdot \vec{i}$$

Again the force changes sign with ***z*** but in the interval $(-d,+d)$ (Fig. 7.6). It is as if the edge core attracts a segment of the screw dislocation.

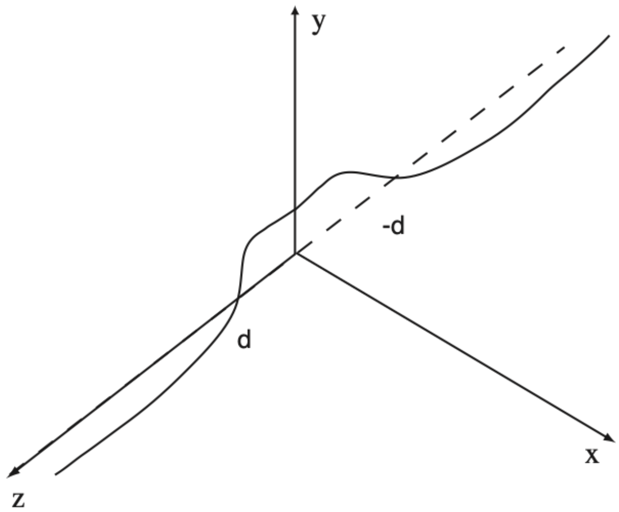

Fig. 7.6 Distortion of the screw dislocation that is attracted by the core of the edge dislocation.

Exercise 3 Interactions between dislocations in f.c.c. metals

1) The reaction gives the formation of a Frank dislocation:

$AB + B\alpha \rightarrow A\alpha$ $\qquad \frac{a}{2}[\bar{1}10] + \frac{a}{6}[1\bar{1}\bar{2}] = \frac{a}{3}[\bar{1}1\bar{1}]$

The Frank dislocation is obtained by combining a dislocation (1) gliding on the plane ADB and another (2) gliding on the plane DBC. The Frank dislocation (3), which is in a dense direction (edge of the tetrahedron), is sessile. When the two dislocations meet, one half remains on the half-plane on the left of the Frank dislocation, and the other continues like closing a zipper on the other half-plane. The results are shown in Fig. 7.7a and 7.7b: there are two possibilities, and they produce either a segment of an intrinsic loop, i.e., an interstitial loop (if looking in the direction of the Burgers vector, the line vector goes from right to left) or a segment of an extrinsic loop (vacancy loop) vice versa.

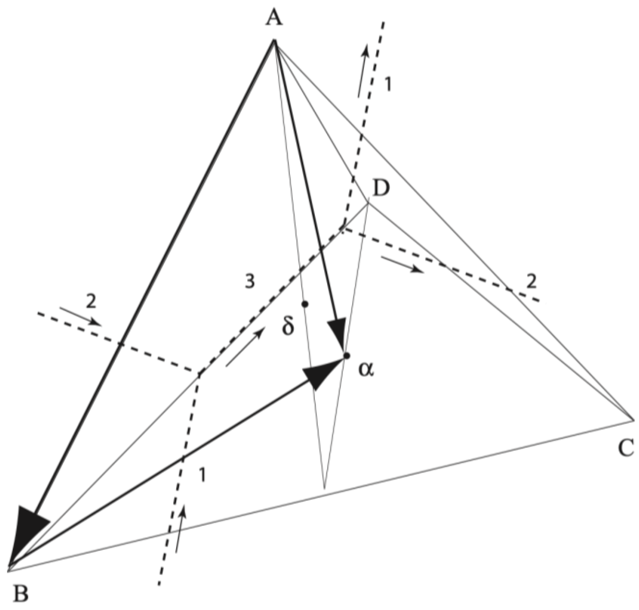

Fig. 7.7a Representation of the Thompson tetrahedron and the formation of a Frank dislocation. Dislocation 3 forms on an edge with a “zipper” mechanism. The elongation of 3 is half of the original dislocations 1 and 2. In this case, segment 3 forms an intrinsic loop.

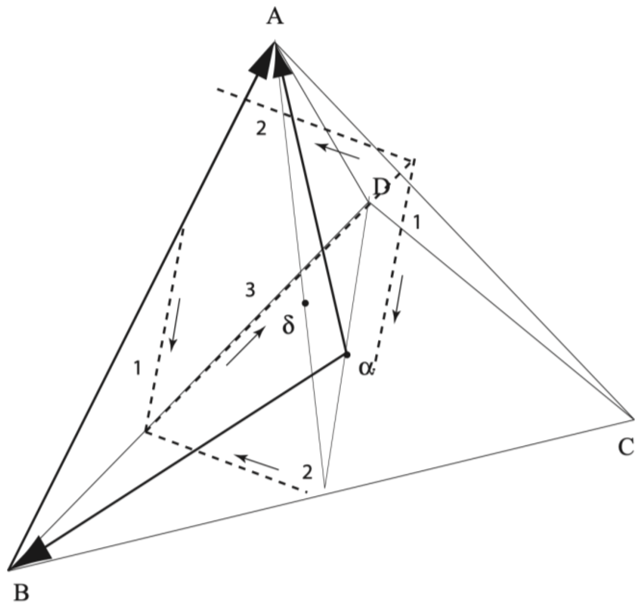

Fig. 7.7b. In this case, segment 3 forms an extrinsic loop.

The reaction between two perfect dislocations gives the formation of a Lomer lock. The resulting perfect dislocation (Lomer) is not on the proper slip plane since its Burgers vector is of BC type and its slip plane is (010) type. It is, therefore, sessile.

$$BD + DC \rightarrow BC \qquad \frac{a}{2}\left[0\bar{1}\bar{1}\right] + \frac{a}{2}[110] = \frac{a}{2}\left[10\bar{1}\right]$$

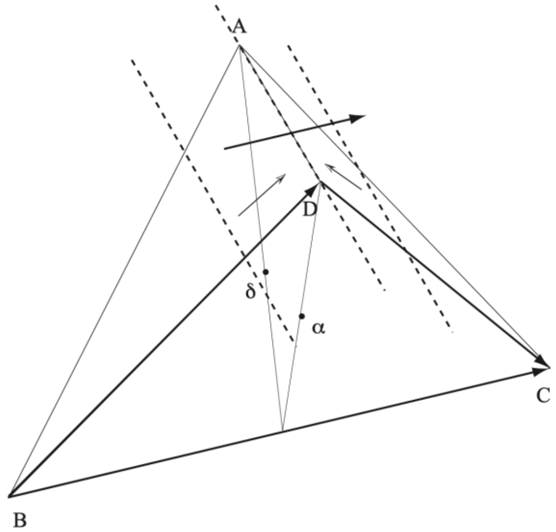

Fig. 7.8 Formation of a Lomer dislocation

2) The formation of a Lomer-Cottrell lock (also called stair-rod) is given by the reaction between two Shockley partial dislocations bordering the stacking faults (green dashed areas) produced by these dissociations:

$$CD \rightarrow C\beta + \beta D \quad \text{and} \quad DB \rightarrow D\gamma + \gamma B$$

The reaction can thus be sketched by $CD + DB = C\beta + \beta D + D\gamma + \gamma B \rightarrow C\beta + \beta\gamma + \gamma B$

$$\frac{a}{2}\left[\bar{1}\bar{1}0\right] + \frac{a}{2}[011] = \frac{a}{6}\left[\bar{1}\bar{2}1\right] + \frac{a}{6}\left[\bar{2}\bar{1}\bar{1}\right] + \frac{a}{6}[112] + \frac{a}{6}\left[1\bar{2}\bar{1}\right] \rightarrow \frac{a}{6}\left[\bar{1}\bar{2}1\right] + \frac{a}{6}\left[\bar{1}01\right] + \frac{a}{6}\left[\bar{1}21\right]$$

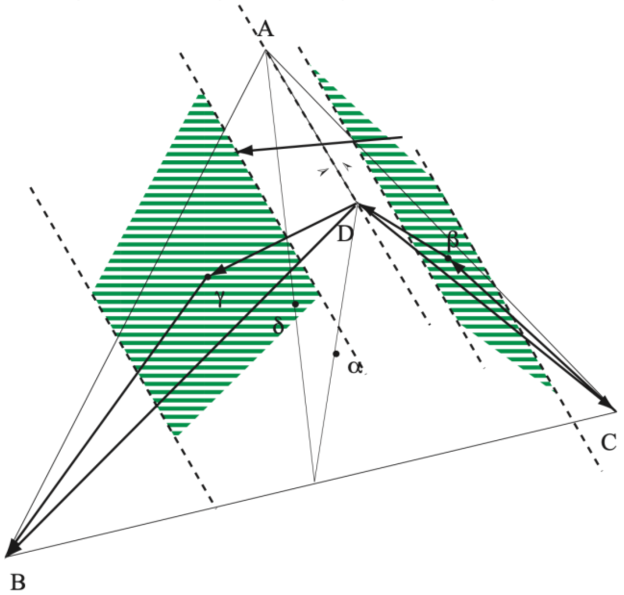

Fig. 7.9 Formation of a Lomer-Cottrell dislocation lock